

**Paper Title:**   **STATIC AND STABILITY ANALYSIS OF LONG-SPAN CABLE-STAYED STEEL BRIDGES**

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# **STATIC AND STABILITY ANALYSIS OF LONG-SPAN CABLE-STAYED STEEL BRIDGES**

By Shuqing Wang and Chung C. Fu, University of Maryland

## **ABSTRACT**

The analysis based on an ideal state as well as the simulation and control to reach it are common issues in the design and construction of a cable-stayed bridge. For a long-span cable-stayed bridge, the huge initial stress accumulated in the pylon and the girder will reduce the overall structure stiffness. Thus, the live load evaluation employing influence lines, which is based on the Muller-Breslau Principle, will no longer be an accurate method. When the main span of the bridge is extended longer, some more critical issues, such as large-deformation effects and stability during construction, will arise.

This paper includes a feasibility study of a steel cable-stayed bridge with a main span of 1,088 meters. Methodologies for defining the final ideal state, the backward and forward analysis for construction simulation and control, the geometric nonlinearity together with the static stability analysis are discussed briefly. Some features of the Visual Bridge Design System (VBDS), which is adopted as the system for conducting the analysis of the 1,088 meter-cable-stayed bridge, are introduced.

The results of the analyses, including the final ideal state, the live load envelope with the consideration of the nonlinear effects of the initial stresses and cable sags, and the stability due to lateral wind and construction loads are summarized. The aerodynamic stability of the structure for flutter and vortex-induced oscillations is not included in this study.

## STATIC AND STABILITY ANALYSIS OF LONG-SPAN CABLE-STAYED STEEL BRIDGES

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### INTRODUCTION

The cable-stayed bridge has been developing rapidly since World War II, and becomes one of the most competitive types of bridges for main spans ranging from 300 to 600 meters. The Normandy bridge (865 meter, 1995, France) and the Tatara bridge (890 meter, 1999, Japan) showed the potential to compete with the suspension bridge in the lower end of its rational span range. For some soft soil bridge sites, on which building of the anchor will dramatically increase the overall cost, a long-span cable-stayed bridge would be the first candidate. It is feasible to build a cable-stayed bridge with a main span as long as 1,200 meters. The results of some feasibility studies on building a cable-stayed bridge with a main span over 1,000 meters motivated some huge bridge projects in Southeast Asia.

In the analysis of a cable-stayed bridge, if no extra adjustment forces are applied to the cable, most of the structural weight of the girder will be distributed along the girder. As the span increases, the internal force and the displacement distribution along the girder will be huge, but the stress of the cable will be far below the allowable stress of the high-strength strands. It is common sense to transfer the girder weight to the pylon by adjusting the cable stresses. How high the stress of each cable should be in the complete stage is the "ideal state" issue of a cable-stayed bridge. The "ideal state" of a cable-stayed bridge can be defined as the minimized total bending energy accumulated along the girder. The dominant issue of the design and build of a cable-stayed bridge is to compute and achieve the ideal state. In order to guide the construction of each erection stage, backward analysis is commonly adopted, in which the bridge is disassembled stage by stage from the ideal state until just before the first pairs of cables are jacked. The forward analysis starting from any construction stage will predict the states in the successive stages by simulating the actual construction procedures. The defining of the ideal state, the backward and forward analyses are two common issues in analysis of a cable-stayed bridge.

As the main span length increases, the geometric nonlinear effect can no longer be ignored. The initial stress effects (also known as P-Delta effects) should be taken into consideration first, which in turn influences the ideal state, the forward and backward analyses, and especially the live load analysis. Because the huge initial stress (axial pressure) accumulated in the pylons and the girder after the bridge is built, the stiffness of the structure will decrease. Loading on influence lines to determine the extreme value will not work due to the nonlinear effects. Reapplying the vehicle loads obtained from live loading on the structure and analyzing with the consideration of initial stress are more practical for calibrating the extreme values, in which the modified influence lines are computed from applying the unit concentrated forces on the structure with the initial stress considered.

Use of the effective Young's modulus of the cable (Ernst, 1965 and Tang, 1971 and 1972) in the analysis model is widely practiced to solve the sag effects of long cables. For a longer span, even the effective Young's modulus should be computed by iteration according to the cable stress in any particular stage.

For analysis of spans over 600 meters, large deformation should be taken into consideration. Based on the large deformation but small strain assumption, the global equilibrium equation can be established together with the consideration of the initial stress effects (Zienkiewicz, 1977). In addition to the linear stiffness, the tangential stiffness of a geometric nonlinear structure will include the geometric stiffness and the large deformation stiffness. In practice, the modified Newton-Raphson method is used to solve the nonlinear equations, in which the loads are applied incrementally on the structure and the Newton-Raphson iterations are performed in each load step.

Stability becomes a critical factor for building a long-span cable-stayed bridge. Both lateral and vertical stability analyses in the maximum cantilever stage and in the service state should be conducted. After the initial stresses are determined, the critical load can be obtained from solving the eigenvalue problem (Tang, 1976 and Ermopoulos, 1992). However, the bifurcation point gives just the upper limit of the stability since a perfect stability problem rarely happens in actual engineering situations. Taking the full tangential stiffness and increasing the load step-by-step are necessary. Thus, stability analysis with consideration of large deformation effects can be the same process as static geometric nonlinear analysis.

However, the load cases for stability analysis should be selected to reflect the nature of the structure. The loads should be able to increase step by step till the deformation has become unreasonably large, or the whole tangential stiffness reaches zero. For example, if the lateral stability under construction loads is of concern, the lateral wind loads, as the minor loads, should be applied on the structure, and the construction loads, as the major loads, should be increased step by step. The level of the major loads at which the structure fails, is the critical load of the stability analysis.

It is shown from the sample in this paper that the full tangential stiffness of a long-span cable-stayed bridge hardly reaches zero. It is reasonable that a change in the configuration would keep the structure stable. Thus, geometric nonlinear analysis only is not sufficient. The material nonlinearity must also be considered. Even the aerodynamic stability analysis is unavoidable.

Many commercial FEA packages are available for solving each particular problem in the cable-stayed bridge. However, the following major reasons make it necessary to develop a special purpose system dedicated to the analysis of cable-stayed bridges: (1) it has to deal with changing of the analysis model to simulate the stage changing, (2) there is a huge amount of computing in accumulating or enveloping the results from different stages and different load cases, (3) the demands to restore the stress and deformation state to any particular stage before conducting nonlinear analysis for any load case (for example, the live load extreme evaluation including the influence line evaluation and the iteration of the vehicle loads should be based on the ideal state or the actual complete state of the bridge), and (4) the demands of some special functions to help the engineer to define the ideal state.

Firstly in this paper, the theories involved in the above issues in the long-span cable-stayed bridge analysis will be briefly discussed. Secondly the VBDS, a sophisticated bridge spatial analysis system, will be introduced. Finally, as an example for employing VBDS, the analysis of a long-span cable-stayed bridge with a main span of 1,088 meters is introduced. The aerodynamic stability of the structure for flutter and vortex induced oscillation is not included in this study.

## THE APPROACH TO THE IDEAL STATE IN COMPLETE STAGE

Distributing the structural weight of the girder up to the pylon by the stayed cables, causing the girder to act like a multi-span continuous girder, is the principle for extending the span of a cable-stayed bridge. If only the girder is considered, it is easy to conclude that the ideal state of a cable-stayed bridge is that the total bending energy accumulated along the girder is minimum. In practice, it is equivalent to adjusting the moment of the girder at the anchor to zero (or even negative) or the vertical displacements at the anchor to zero.

If the pylon has to be considered together with the girder, certainly having no longitudinal displacements or no bending moment is perfect. Since most bridges are not symmetrical about pylons, bearing a minor moment is unavoidable.

The moment and displacement distribution along the girder and the pylon can reach the ideal state by adjusting the cable stresses. The moment or the displacement of an ideal state,  $\mathbf{Z}$ , can be written as

$$\mathbf{Z} = \{z_1 \quad z_2 \quad \dots \quad z_n\}^T \quad (1)$$

where  $n$  is the total number of the targets that need to be satisfied and  $T$  stands for the transformation of a matrix or a vector. The approach to the ideal state is to make equation (1) as close to a designated value as possible. The square error minimizing method is one of the most effective ways to obtain the optimal  $\mathbf{Z}$ , in which the result cable stresses  $\mathbf{S}$  can be written as

$$\mathbf{S} = \{s_1 \quad s_2 \quad \dots \quad s_m\}^T \quad (2)$$

where  $m$  is the total number of the cable to be tuned.

By analyzing the response of the unit prestress applied at each pair of the tuning cables, the influence value of all the targets can be obtained. When  $m$  rounds of analysis are done, the influence matrix  $\mathbf{A}$  can be written as

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix} \quad (3)$$

where  $a_{ij}$  is the response at target  $i$  by prestressing the unit stress at cable  $j$ . Thus, their relationship can be written as

$$\mathbf{A} \times \mathbf{S} = \mathbf{Z} \quad (4)$$

If the number of the cables to be tuned is the same as the number of targets, setting  $\mathbf{Z}$  to the designated target values, the cable stresses can be obtained accordingly by solving the linear equation (4). In this case, engineering experience is required in selecting the cables and the targets. A bad or contradictory tuning of cables and targets may cause equation (3) not to be a diagonal dominant matrix or equation (4) to be in ill condition. In this method,  $m$  cannot be greater than  $n$ . If, as in most cases,  $m$  is less than  $n$ , the cable stresses can be optimized so that the error of the target value and the designated state is minimum.  $\mathbf{D}$ , which has the same form as  $\mathbf{Z}$ , is the designated target value.  $\mathbf{E}$ , the error between  $\mathbf{D}$  and  $\mathbf{Z}$ , can be written as

$$\mathbf{E} = \mathbf{D} - \mathbf{Z} \quad (5)$$

The optimization of the cable stress is to minimize  $\Omega$ , the square of  $\mathbf{E}$ . As the definition,  $\Omega$  can be written as

$$\Omega = (\mathbf{D} - \mathbf{Z})^2 \quad (6)$$

From the variational principle, it is known that the condition to have  $\Omega$  minimized is

$$\frac{\partial \Omega}{\partial \mathbf{S}_i} = 0, i = 1, 2, 3, \dots, m \quad (7)$$

By using the matrix differential and considering equations (6) and (4), the following equation can be obtained.

$$\mathbf{A}^T \mathbf{A} \times \mathbf{S} = \mathbf{A}^T \mathbf{D} \quad (8)$$

After solving  $\mathbf{S}$  from the linear equation group in equation (8), the optimized target value will be known from equation (4).

In practice, the following procedures are common in approaching the ideal state at service stage.

- (a) Select all the cables to be tuned
- (b) Perform the static analysis under structural weight and the superimposed dead load
- (c) Select the negative displacement of the girder at each anchor in (b) as  $\mathbf{D}$ . This step varies in different situations.
- (d) Evaluate  $\mathbf{S}$  as above
- (e) Similar to prestressing loads, reapply  $\mathbf{S}$  on the structure and perform the analysis
- (f) The sum of (b) and (e) is the ideal state at complete stage

## THE BACKWARD AND FORWARD ANALYSIS

Assuming that the ideal state at the complete stage as defined above is really what is reached at the end of the segment-by-segment construction, disassembling of the bridge stage by stage starting from the complete stage will trace back to each state of the erection process. The backward analysis results are the guidance for its forward erection and stressing. But it has to be noted that it is impossible for a actual forward stage to reach exactly the state the backward analysis obtained. This can be understood at least from the fact that the closure segment will not be in the zero stress state after the superimposed dead load is removed in backward analysis, while in reality it is in the zero stress state after closure.

Unlike backward analysis, the forward analysis based on the actual state of any construction stage can predict the state when the bridge closes in the middle span. This prediction is very important for cable tunings at any stage. Since retuning every pair of cables will increase labor onsite dramatically and hence slow the pace, usually only the newly installed pairs of cables are jacked according to the analysis results backward to that stage. If, however, tuning one pair of cables cannot keep the state of the bridge in control, retuning of multiple cables will be required. The retuning is required at least in the complete stage.

To simulate the removal and the installation of some components of a cable-stayed bridge, a dedicated analysis program is required. The backward analysis can be treated as follows:

- (a) Apply the negative nodal forces of the removal components on the structure after removal
- (b) The sum of (a) and the state before removing the components is the state after removal.

The forward analysis can be treated as follows:

- (a) Analyze the new structure with the weight of the newly installed components
- (b) Analyze the prestressing of the newly installed cables, if applicable
- (c) The sum of (a), (b) and the state before the installation is the state after erection.

### THE GEOMETRIC NONLINEAR ANALYSIS

The geometric nonlinear effects of cable-stayed bridges include (1) P-Delta effect or the initial stress, (2) sag effects of long cables, and (3) the large deformation (Fleming, 1979 and Abdel-Ghaffar, et al, 1987). The P-Delta effect should be considered in most cable-stayed bridges, at least in the live load analysis since the huge axial forces are accumulated along the girder and the pylon after the bridge is complete. The other two effects should also be considered for a long-span cable-stayed bridge (Fleming, 1979).

According to the finite element method, the principle of solving the geometric nonlinear problem can be stated as follows (Zienkiewicz, 1977).

In spite of the large displacement of the system, the equilibrium conditions between internal and external 'forces' have to be satisfied as

$$\boldsymbol{\psi}(\mathbf{a}) = \int_V \bar{\mathbf{B}}^T \boldsymbol{\sigma} dV - \mathbf{f} = 0 \quad (9)$$

where  $\boldsymbol{\psi}$  is the sum of internal and external generalized forces,  $\mathbf{a}$  is the displacements prescribed at a finite number of nodal,  $\mathbf{f}$  is the external forces acting on the structure,  $\boldsymbol{\sigma}$  is the stress, and  $\bar{\mathbf{B}}$  defines the relationship between the strain and the nodal displacements as

$$\boldsymbol{\varepsilon} = \bar{\mathbf{B}}\mathbf{a} \quad (10)$$

The bar suffix represents the relationship that is no longer linear since the large displacement is considered, in which the strain is now dependent on the displacement as

$$\bar{\mathbf{B}} = \mathbf{B}_0 + \mathbf{B}_L(\mathbf{a}) \quad (11)$$

If, as assumed, strains are reasonably small, it can still use the general elastic relation as

$$\boldsymbol{\sigma} = \mathbf{D}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0) + \boldsymbol{\sigma}_0 \quad (12)$$

where  $\mathbf{D}$  is the elastic matrix represented the relationship between the stress and the strain,  $\boldsymbol{\varepsilon}_0$  is the initial strain and  $\boldsymbol{\sigma}_0$  is the initial stress.

In order to solve  $\mathbf{a}$  at known  $\mathbf{f}$ , Newton-Raphson process can be adopted. Learning the relation between  $d\mathbf{a}$  and  $d\boldsymbol{\psi}$  is required. Thus, taking appropriate variations of equation (9) with respect to  $d\mathbf{a}$ , equation (13) can be derived as

$$d\boldsymbol{\psi} = (\mathbf{K}_0 + \mathbf{K}_\sigma + \mathbf{K}_L)d\mathbf{a} = \mathbf{K}_T d\mathbf{a} \quad (13)$$

in which  $\mathbf{K}_T$  is the total tangential stiffness matrix,  $\mathbf{K}_0$  represents the usual, small displacements stiffness matrix, i.e.,

$$\mathbf{K}_0 = \int_V \mathbf{B}_0^T \mathbf{D} \mathbf{B}_0 dV \quad (14)$$

$\mathbf{K}_\sigma$  is known as initial stress matrix or geometric matrix. It depends on the stress and can be written as

$$\int_V d\mathbf{B}_L^T \boldsymbol{\sigma} dV \equiv \mathbf{K}_\sigma d\mathbf{a} \quad (15)$$

$\mathbf{K}_L$  is known as the initial displacement matrix or large displacement matrix and given by

$$\mathbf{K}_L = \int_V (\mathbf{B}_0^T \mathbf{D} \mathbf{B}_L + \mathbf{B}_L^T \mathbf{D} \mathbf{B}_L + \mathbf{B}_L^T \mathbf{D} \mathbf{B}_0) dV \quad (16)$$

The procedures of the Newton-Raphson iteration method to solve this nonlinear problem are

- (a) obtain the first approximation of the displacement as  $\mathbf{a}^0$  based on the elastic linear solution

(b)  $\boldsymbol{\psi}^0$ , the unbalance force or remained force, is found using equation (9) with appropriate definition of  $\bar{\mathbf{B}}$  and stress as given by equation (12)

(c)  $\mathbf{K}_T^0$  is established, and

(d) compute the correction of the displacement,  $\Delta \mathbf{a}^0$ , by solving the following equation

$$\mathbf{K}_T^0 \Delta \mathbf{a}^0 + \boldsymbol{\psi}^0 = 0 \quad (17)$$

and repeat (b), (c) and (d) until  $\boldsymbol{\psi}^n$  becomes sufficiently small.

In the nonlinear analysis of a cable-stayed bridge, an incremental method is usually adopted together with the Newton-Raphson method, in which the loads are scheduled to act on the structure incrementally. It is obvious that the P-Delta effect is reflected from  $\mathbf{K}_\sigma$  and will automatically be taken into consideration in the above process.

Since the unbalanced forces cannot reach zero exactly, the remainder should be accumulated to the analysis of the next successive load or stage in the forward and backward analyses, otherwise the error could be significant. No matter whether forward or backward, the accumulated stress and displacement up to the previous stage or previous load case should be taken into account in computing equations (15) and (16) accordingly. Lack of such a feature makes many commercial packages not practical in cable-stayed bridge analysis.

Sag nonlinearity of a long cable is reduction of the actual stiffness along the straight line between the two anchors due to the sag and related to the current stress in the cable. If the cables are simply modeled as straight truss elements, Ernst formula (Ernst, 1965) is usually adopted to calculate their equivalent stiffness (Tang, 1971 and 1972). However, its drawbacks are (1) the error will be significant for a long and low stress level cable and (2) the coupling of stiffness and stress makes the solving process not unique. It can be bypassed either by adopting a special cable element as provided in some commercial package or simply sub-meshing the cable.

## THE STABILITY ANALYSIS

If the large deformation is ignored, the total tangential stiffness in equation (13) will have only  $\mathbf{K}_0$ , the elastic, small displacement stiffness matrix and  $\mathbf{K}_\sigma$ , the initial stress stiffness matrix. Since the axial forces along the pylon and the girder are pressure,  $\mathbf{K}_\sigma$  will reduce to  $\mathbf{K}_T$ . If the loads that cause the initial stress, usually the structural weight and cable stressing, keep increasing, a critical point will be reached, at which the determinant of the total stiffness matrix is zero.

Such a bifurcation stability problem can be solved as an eigenvalue problem (Tang, 1976 and Ermopoulos 1992). In actual situations, however, it rarely happens due to the flaws in building the structure.  $\mathbf{K}_L$  should also be considered and the full Newton-Raphson process is required. In some typical situations, it is easy to understand. For example, the transverse stability due to the live load of a vertically stayed cable bridge under transverse wind loads will be enhanced after the deck moves laterally away from the centerline. Not only the tension, the positive  $\mathbf{K}_\sigma$ , but also the laterally sloped geometry,  $\mathbf{K}_L$  of the cables will enhance the lateral stiffness.

Again, the stability analysis of a long-span cable-stayed bridge can be combined with its nonlinear analysis. The analysis of a long-span cable-stayed bridge with a main span of 1,088 meters, however, shows that the statically geometric nonlinear stability analysis is not sufficient. The total tangential stiffness, with  $\mathbf{K}_L$  included, hardly reaches zero. This suggests that aerodynamic stability analysis and the geometric plus material nonlinear analysis are required (Ren, 1999).

## SOME FEATURES OF VBDS

VBDS, Visual Bridge Design System, is an integrated approach to bridge modeling, analysis and design. Its objective is to provide engineers with sophisticated structural analysis facilities suitable for any type of

bridge and the most commonly used construction methods. The cable-stayed bridge is one of the most suitable types of bridge to use this package.

Its development fully employs state-of-the-art software technologies, such as object-oriented programming, database connectivity, visualization, graphical user interface, etc. It consists of a preprocessor for modeling bridges with the support of AutoCAD, the finite element analysis kernel, a postprocessor and some bridge type oriented design systems. All the subsystems are integrated together by sharing the same database. Some special features for cable-stayed bridge modeling and analysis are listed below.

- (a) Modeling a cable-stayed bridge with spatial frame, truss and shell elements by graphical support of AutoCAD
- (b) Unlimited construction stages with unlimited load cases with forward and backward analyses committed
- (c) Automatic cable tuning and optimization towards ideal state
- (d) Geometric nonlinear analysis including sag of cable, initial stress and large deformation in forward and backward analyses
- (e) Natural mode and stability analyses
- (f) Live load analysis with consideration of the sag effect and initial stress during influence line computation and automatically reanalyzing the large deformation effects by reapplying vehicle loads on the structure
- (g) Property sensitive analysis

By using the preprocessor subsystem of VBDS, a cable-stayed bridge can be modeled with the assistance of any graphical capabilities available in AutoCAD. For example, the user can draw the centerlines of the pylon, and then invoke the function to mesh the lines to 3D frame elements. When the model is created, some other functions can be used to define each stage and the tuning cables. All the modeling assistance functions are integrated with AutoCAD through several pull-down menus. It also gives the user a shortcut to switch quickly between the native AutoCAD menu and the preprocessor menu.

#### **ANALYSIS OF A CABLE-STAYED BRIDGE**

A proposed steel cable-stayed bridge project is taken as an example to illustrate issues in this paper. The elevation profile, the typical cross section of the steel box girder, and the concrete pylon are shown in Figures 1-3. Two different types of diaphragms are placed at an interval of 4 meters. The diaphragms at the anchor positions are different from that at other positions. Extra counterweights are placed from the first pier to the second auxiliary pier. The analysis focuses on only three stages: (1) the service stage, (2) maximum single-cantilever stage and (3) maximum dual-cantilever stage, and includes the following goals:

- (a) ideal state and live load analyses of the service stage
- (b) static analysis of the other two stages
- (c) static wind load analysis and the stability analysis
- (d) geometric nonlinear effects analysis.

The steel girder and the pylon are modeled as 3D frame, the diaphragm at the anchor position is modeled as rigid body and the cable is modeled as 3D truss. Totally, the model is meshed with 1032 elements and 1035 nodes. VBDS is employed in the analysis. ANSYS is also employed for checking some analyses. Since the ideal state cannot be defined by cable tuning automatically in ANSYS, the analysis conducted in ANSYS includes structural weight at service stage without any stressing of the cable. Thus, it can only serve for the purpose of checking.

Figure 4 shows the ideal state approached by automatic cable tuning. Figure 5 shows the live load stress envelope. Table 1 compares the extreme displacements due to live load with and without the geometric nonlinear effects. The nonlinear analysis of the vehicle loads obtained according to live loading shows that the initial stress accumulated along the flat arch-like girder will increase the girder stiffness if the large deformation is considered.

The wind pressure is designated according to the bridge site. It varies at different altitudes along the pylon. With regard to the longitudinal connection between girder and the pylon, three alternatives are studied. The first, the recommended one, is that the girder is restrained with one pylon only. The second is when the girder is not restrained with any pylon, the third is restrained with two pylons. The girder is restrained in both vertical and lateral directions with the pylons in all three alternatives. Figure 6 shows the

displacements of the second system at service stage due to the longitudinal static wind load. Figure 7 shows the lateral displacements of the pylon at the maximum dual-cantilever stage due to lateral wind load.

Several different loading patterns are taken in the stability analysis of this bridge. Table 2 lists the load patterns and critical load of the stability analysis. In the six loading patterns, only the increment of construction load, which includes a 100-ton crane at the end of the girder and a uniform load of 1 ton/meter at the maximum single-cantilever stage, shows the coupling of bending in vertical and in lateral directions. Figure 8 shows the vertical and lateral displacements when the construction loads increase to 46 times the above construction load, while the lateral wind load remains unchanged. The stability analysis also shows that the structure at the stage when its main span is ready to close is more vulnerable than at the stage when its side span reaches to the second auxiliary pier. Although the results of these six loading patterns show that the structure has sufficient stability against live loads, wind load, construction load and the structural weight, the full nonlinear ultimate analysis (Ren, 1999), in which the material nonlinearity is also considered, and the aerodynamic stability analysis is still required.

## CONCLUSIONS

The final ideal state definition and approaching, the forward and backward analysis for construction simulation and control are preliminary tasks in the analysis of a cable-stayed bridge. It involves the optimization of the internal forces and displacements by adjusting cable stresses and the interactions between erection stages. A dedicated program is required to conduct the analysis.

The geometric nonlinear effects due to the initial stresses in the girder and pylons, the cable sags and the large deformation must be considered for the long span cable-stayed bridges. For the static analysis of any erection stage, this can be done by conducting the full geometric nonlinear analysis. For the live load envelope, a practical method is consideration of these effects during the influence line evaluation stage and re-analyzing the extreme live loads on the geometric nonlinear structures to correct the results. In this study, since the girder is mildly arched, the extreme vertical displacement at the center of the main span with consideration of geometric nonlinear effects is even less than that of the linear structure.

The static wind load stability analysis during the erection of a cable-stayed bridge shows that the structure has sufficient stiffness to resist the wind and the construction loads. The changed geometry of the bridge, to some extents, enhances the overall stiffness, if the large deformation is also considered in the stability analysis. However, this does not prove the bridge has a good wind resistance performance. The aerodynamic stability analysis and wind tunnel testing, which are not covered in this study, are necessary to approve the feasibility of building a super-long cable-stayed bridge.

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- FIGURE 1 – The elevation profile of the example steel cable-stayed bridge
- FIGURE 2 – The Typical Cross Section of the Steel Box Girder of the Example Steel Cable-Stayed Bridge (Unit:mm)
- FIGURE 3 - Transverse and Longitudinal View of the Pylon with Its Cross Sections of the Example Steel Cable-Stayed Bridge (Unit:cm)
- FIGURE 4 - The ideal state moment (kN-m) and displacement (mm) distribution
- FIGURE 5 - The live load stress envelope (MPa, on top of the girder and side-span side of pylon)
- FIGURE 6 - Displacements due to longitudinal static wind load (mm) of the non-restrained system
- FIGURE 7 - Lateral displacements of the pylon at maximum dual-cantilever stage due to static wind load in lateral direction (mm)
- FIGURE 8 - The vertical (top) and the lateral (bottom) displacements (m) of the girder when the construction loads increased to 46 times of the normal construction loads at the maximum single-cantilever stage
- Table 1- Extreme displacements due to live loads (mm)
- Table 2- Loading patterns and the critical loads in stability analysis





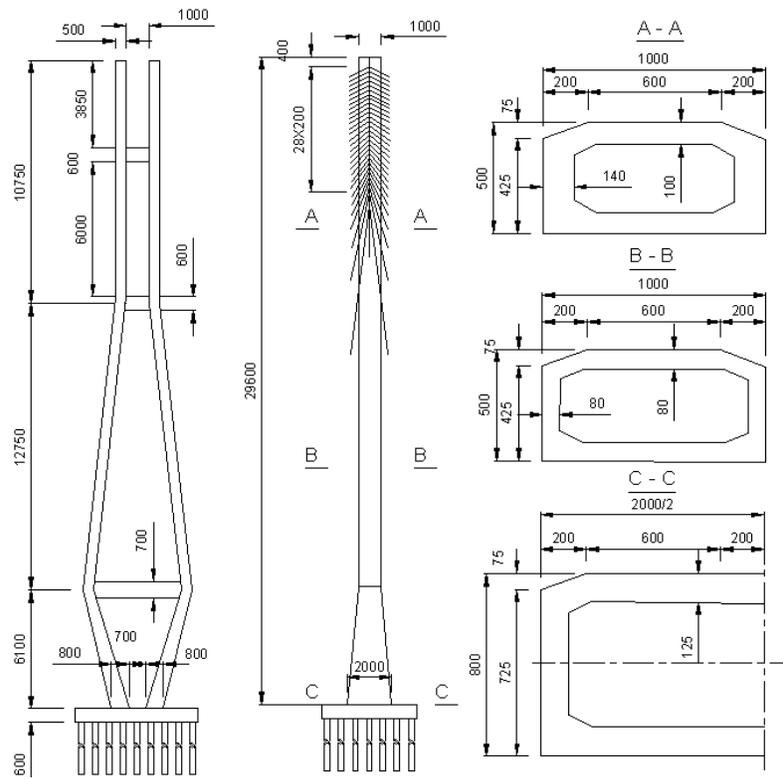


FIGURE 3 - Transverse and Longitudinal View of the Pylon with Its Cross Sections of the Example Steel Cable-Stayed Bridge (Unit:cm)

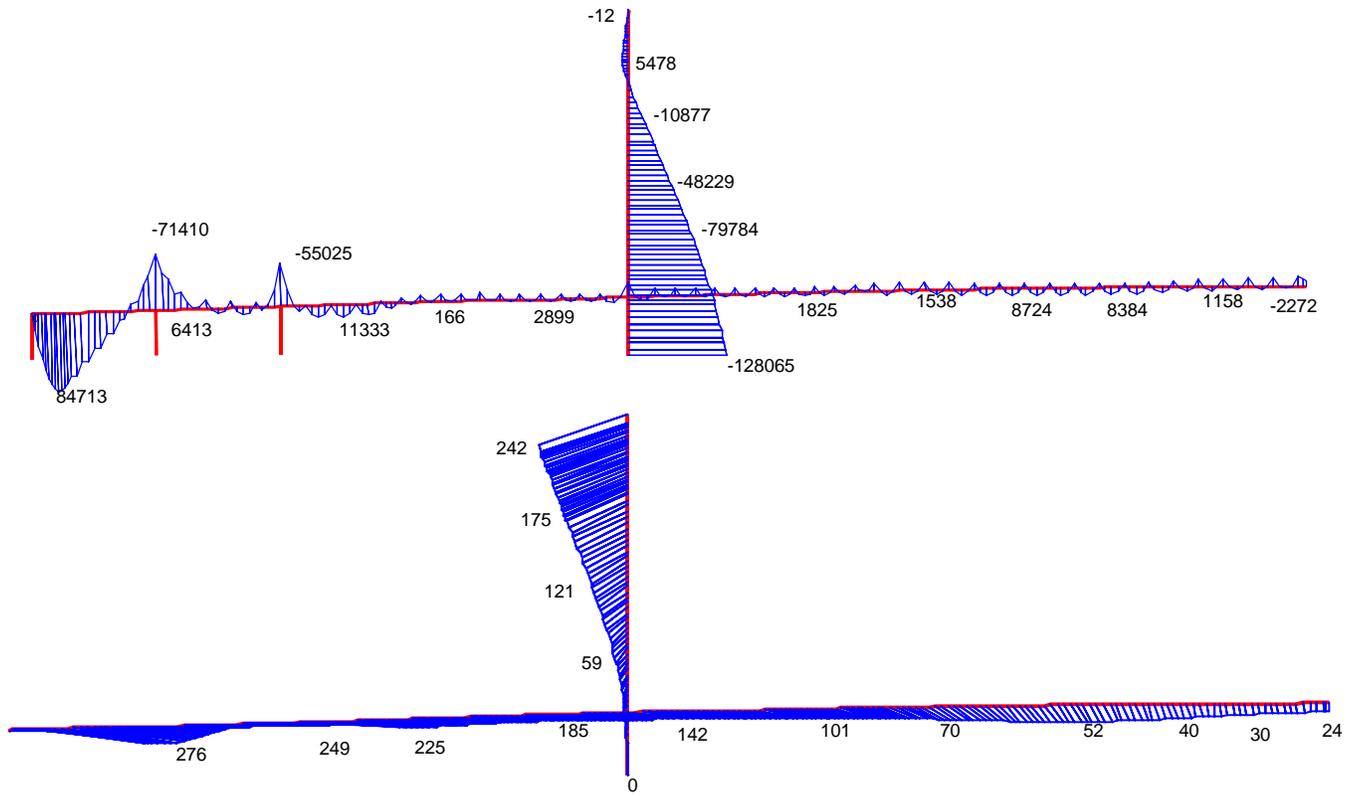


FIGURE 4 - The ideal state moment(kNm) and displacement(mm) distribution

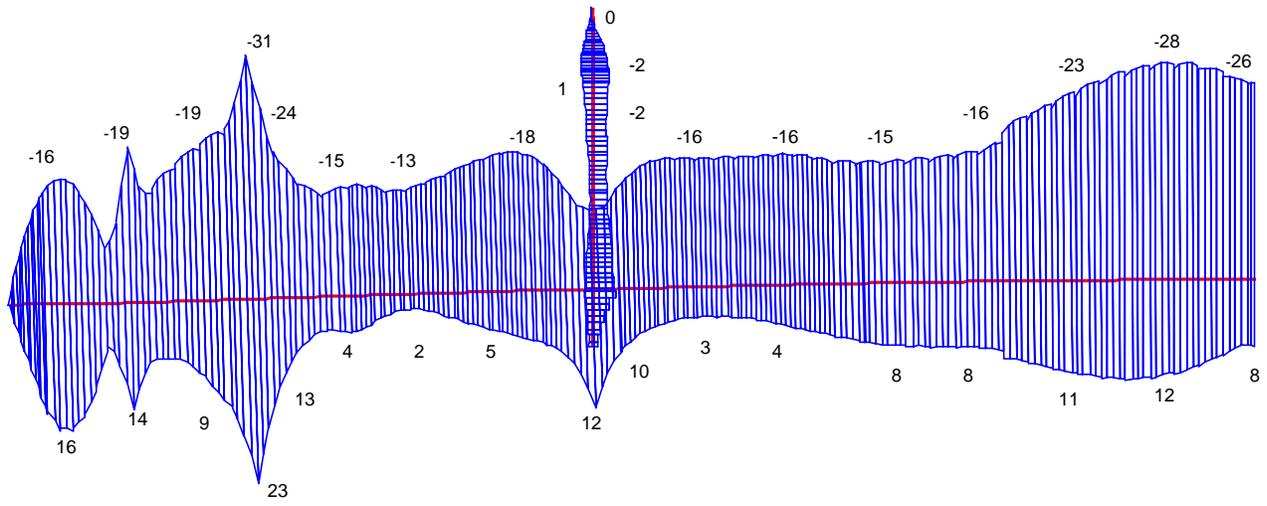


FIGURE 5 - The live load stress envelope (Mpa, on top of the girder and side-span side of pylon)

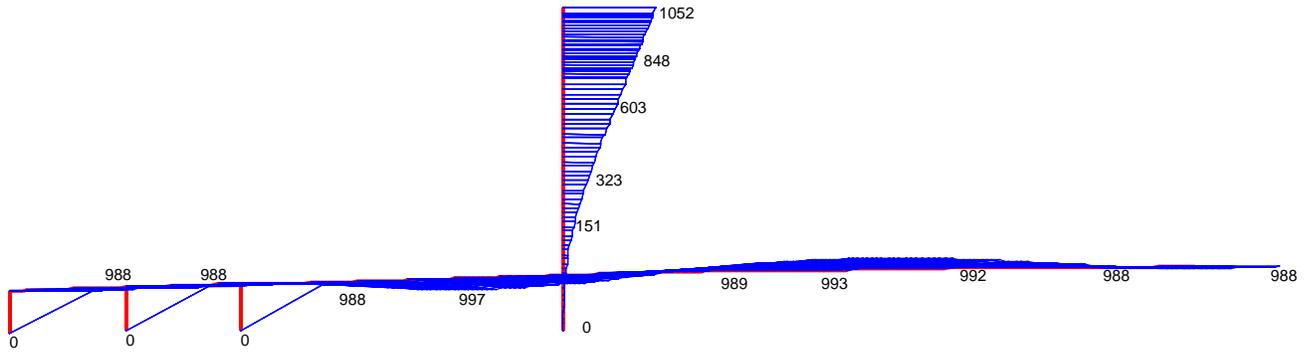


FIGURE 6 - Displacements due to longitudinal static wind load (mm) of the non-restrained system

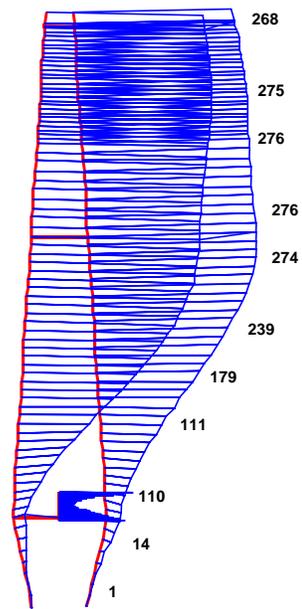


FIGURE 7 - Lateral displacements of the pylon at maximum dual-cantilever stage due to static wind load in lateral direction (mm)

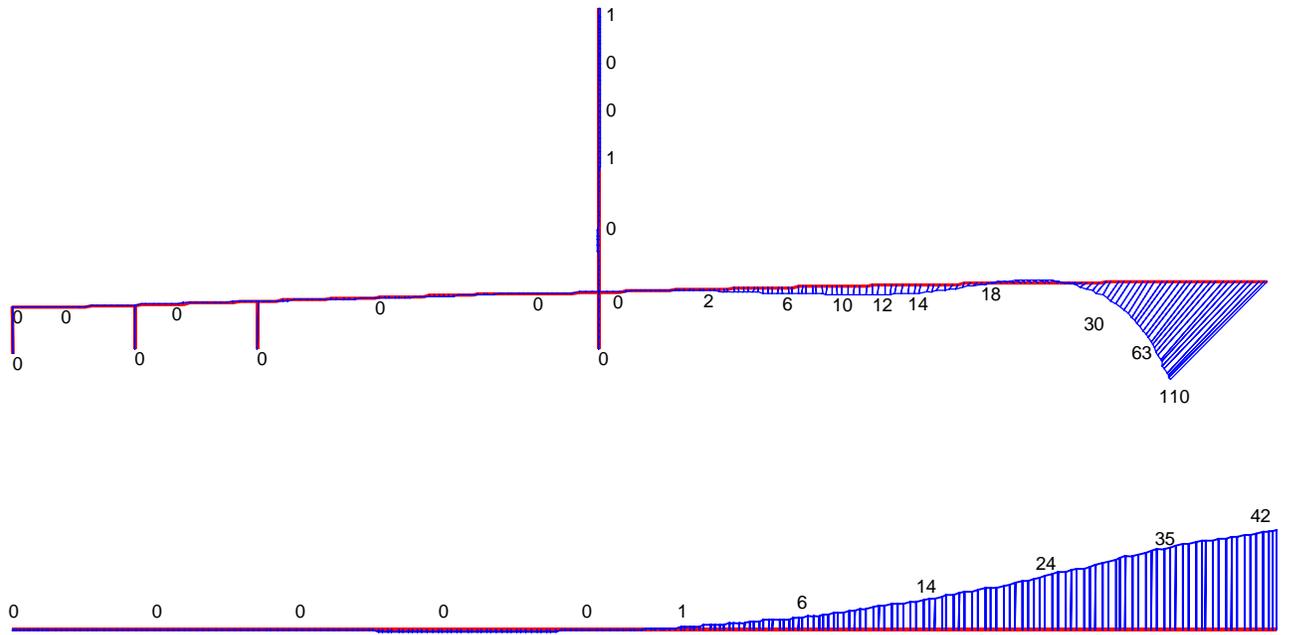


FIGURE 8 - The vertical (top) and the lateral (bottom) displacements (m) of the girder when the construction loads increased to 46 times of the normal construction loads at the maximum single-cantilever stage

Table 1- Extreme displacements due to live loads (mm)

Position	Linear <sup>(1)</sup>	Nonlinear <sup>(2)</sup>
Vertical, in the middle of the main span	1081	935
Longitudinal, on the top of the pylon	263	242

- (1) computed from directly loading on the influence line, which is obtained from applying a unit force on the girder with consideration of the initial stress and cable sag effect.
- (2) Treating the vehicle load of (1) as a load case and applying it on the structure, with the consideration of initial stress, cable sag effect and large deformation during the re-analysis.

Table 2- Loading patterns and the critical loads in stability analysis

Loading patterns	Description	Critical case
At $S_0$ , increase $V$ step by step	To search the live load safety factor without wind interfering at service stage	When the live loads increased up to 40 times of the normal live load, the vertical displacements at the center of the main span abruptly reached 42 meters and the 13 meters at the top of the pylon. The structure, however still maintains some degree of stiffness. No lateral displacement significantly increased.
At $S_0$ , increase $S$ step by step	To search the whole structural weight safety factor without wind interference at service stage	At about 3 times of $S$ , the displacements increase abruptly. No lateral displacement significantly increased.
At $S_1$ plus $W$ , increase $C$ step by step	To search the construction load safety factor with wind interference at maximum dual-cantilever stage	When increased to 240 times of $C$ , the displacements increase abruptly. No lateral displacement significantly increased.
At $S_1$ , increase $W$ step by step	To search the static wind load safety factor at maximum dual-cantilever stage	Still remains in elastic even at 50 times of $W$ , while the lateral displacement at the end of the girder reaches to 7 meters.
At $S_2$ plus $W$ , increase $C$ step by step	To search the construction load safety factor with wind interfering at maximum single-cantilever stage	At 46 times of $C$ , the vertical displacement at the end of the girder increased to over 100 meters accompanied with 42 meters of lateral displacements (Figure 8).
At $S_2$ , increase $W$ step by step	To search the static wind load safety factor at maximum single-cantilever stage without the consideration of the construction load	At 48 times of $W$ , the lateral displacement at the end of the girder increased to over 100 meters.

$S_0$ : the ideal state at service stage (the structural weight, cable tuning and the superimpose dead load)

$S_1$ : the state at the maximum dual-cantilever stage (the structural weight and the cable tuning)

$S_2$ : the state at the maximum single-cantilever stage (the structural weight and the cable tuning)

$S$ : the whole structure weight plus superimpose dead load

$V$ : the live loads that cause the maximum vertical displacement at the center of the main span

$C$ : a 100-ton crane at one or two ends of the cantilever and 1-ton/meter of other construction load

$W$ : the lateral wind load