# TORSIONAL ANALYSIS FOR PRESTRESSED CONCRETE MULTIPLE CELL BOX

**By Chung C. Fu[1], Fellow, ASCE, and Yi Tang[2]**

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**KEYWORDS:** bridge design; bridge analysis; concrete box; prestressed concrete; softened truss model; torsion

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## INTRODUCTION

Concrete is a material that is very strong in compression but weak in tension. When concrete is used in a structure to carry loads, the tensile regions are expected to crack and, therefore, must be reinforced with materials of high tensile strength, such as steel. The concept of utilizing concrete to resist compression and steel reinforcement to carry tension gave rise to the **struts-and-ties model**. In this model, concrete compression struts and steel tension ties form a truss that is capable of resisting applied loads.

The struts-and-ties concept is easily applied to reinforced concrete beams. For example, under bending, the compressive stress in the upper part of a simply supported beam is resisted by concrete in the form of a horizontal strut; the tensile stress in the lower portion is taken by the bottom steel in the form of a horizontal tie. The forces in the concrete and steel must be in equilibrium, and they form a couple to resist the applied bending moment.

The first application of the concept of truss model to members subjected to shear was proposed by Ritter (1899) and Mörsch (1909) in connection with reinforced concrete beams subjected to shear and bending. In their view, a reinforced concrete beam acts similar to a parallel-stringer truss to resist bending

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and shear. Due to the bending moment, the concrete strut near the upper edge serves as the top stringer in a truss and the steel bar near the lower edge assumes the function of the bottom stringer. From shear stresses, the web region would develop diagonal cracks at an angle á inclined to the longitudinal steel. These cracks would separate the concrete into a series of diagonal concrete struts. To resist the applied shear forces after cracking, the transverse steel bars in the web would carry tensile forces and the diagonal concrete struts would resist the compressive forces. The transverse steel, therefore, serves as the tensile web members in the truss and the diagonal concrete struts become the diagonal compression web members.

Although Ritter and Mörsch contributed significantly to the understanding of reinforced concrete structural behavior, their truss models could not explain some types of behavior of reinforced concrete, particularly regarding the so-called contribution of concrete. Researchers did not pursue this line of study until the late 1960s and early 1970s when Nielson (1971) and Lampert and Thurlimann (1971) derived the three fundamental equilibrium equations for shear based on the theory of plasticity. These theories were known collectively as the **plasticity truss model** because they were based on the yielding of steel. In the unified theory for reinforced concrete behavior, the **equilibrium truss model** takes into consideration the equilibrium condition alone. The roles of the compatibility condition and the constitutive laws of materials need to be investigated in the future.

Earlier researchers developed some equations based on a softened truss model for a single cell torsion problem (Hsu, T.T.C., 1988, 1990, 1991a,b, 1993, 1994; Collins and Mitchell, 1991). However, because many concrete bridge superstructures consist of multibox cells, additional equations are created to solve the multicell torsion problem.

## TORSIONAL ANALYSIS FOR MULTIPLE CELL BOX

In early development by others (Collins & Mitchell, 1991; Hsu, 1994), a set of equations is given for solving single cell torsion. A reinforced concrete prismatic member is subjected to an external torque $T$ as shown in Figure 1(a). The external torque is resisted by an internal torque formed by the circulatory shear flow $q$ along the periphery of the cross section. The shear flow $q$ occupies a zone, called the shear flow zone, which has a thickness denoted $t_d$. This thickness $t_d$, or equivalent thickness for a uniform shear stress, is a variable determined from the equilibrium and compatibility conditions. It is not the same as the given wall thickness $h$ of a hollow member. Element A in the shear flow zone [Figure 1(a)] is subjected to a shear stress $\tau_{lt} = q/t_d$ as shown in Figure 1(b).

In bridge engineering, many reinforced concrete bridges consist of multiple box cells. Therefore, a set of simultaneous equations to analyze structural torsion for multiple box cells is needed. In this paper, equations for single cell are developed into ones for multiple cells, and a solution method is provided.

**Equations for Multiple Cell**

Assume a structural section has $N$ cells (Fig. 2). According to restraint condition $\theta = \theta_1 = \theta_2 = \ldots = \theta_N$, a set of simultaneous equations for cell $i$ can be obtained.

**Equilibrium Equations**

A prestressed concrete element, as shown in Figure 3(a), is reinforced orthogonally with longitudinal and transverse (prestressing or nonprestressing) steel reinforcements. The applied stresses on the element have three stress components, $\sigma_l$, $\sigma_t$, and $\tau_{lt}$. The longitudinal steels are arranged in the $l$-direction (horizontal axis) with a uniform spacing of $s_l$. The transverse steels are arranged in the $t$-direction (vertical axis) with a uniform spacing of s [Fig. 3(a)]. After cracking, the concrete is separated by diagonal cracks into a series of concrete struts, as shown in Figure 3(b). The cracks are oriented at an angle $\alpha$ with respect to the $l$-axis. The principal stresses on the concrete strut itself are denoted as $\sigma_d$ and $\sigma_r$. According to the unified theory (Hsu, 1993), after transformation, the governing equations for equilibrium condition are shown as follows

$$\sigma_l = \sigma_d \cos^2\alpha + \sigma_r \sin^2\alpha + \rho_l f_l + \rho_{lp} f_{lp} \tag{1}$$

$$\sigma_t = \sigma_d \sin^2\alpha + \sigma_r \cos^2\alpha + \rho_t f_t + \rho_{tp} f_{tp} \tag{2}$$

$$\tau_{lt} = (-\sigma_d + \sigma_r)\sin\alpha\cos\alpha \tag{3}$$

$$T = \tau_{lt}(2A_0 t_d) \tag{4}$$

where $\sigma_l$, $\sigma_t$, and $\tau_{lt}$ = three homogenized stress components of the composite element [Figure 3(a)]; $\sigma_d$, $\sigma_r$ = concrete stresses in *d*- and *r*-directions, respectively [Figure 3(b)]; $\alpha$ = angle between *l* and *d* axes; $f_l$, $f_t$ = stresses in steel in the *l*- and *t*-directions, respectively; $f_{lp}$, $f_{tp}$ = stresses in prestressing steel in the *l*- and *t*-directions, respectively; $\rho_l$ and $\rho_t$ = steel ratio in the *l*- and *t*-directions, respectively; $\rho_{lp}$, $\rho_{tp}$ = prestressing steel ratio in the *l*- and *t*-directions, respectively; *T* = external torque; $A_0$ = cross-sectional area bounded by the center line of the shear flow zone; and $t_d$ = shear flow zone thickness.

It should be noted that, for multiple cell box under pure torsion, $\sigma_l = \sigma_t = \sigma_r = 0$ and, assuming a structural section has *N* cells, we can then obtain a set of simultaneous equations for cell *i*

$$\tau_{lti} = -\sigma_{di} \sin\alpha_i \cos\alpha_i \tag{5}$$

$$T_i = \tau_{lti}\left(2A_{0i}t_{di}\right) \tag{6}$$

**Compatibility Equations**

Similarly, the governing equations for compatibility condition based on the unified theory (Hsu, 1993) are shown as follows:

$$\varepsilon_l = \varepsilon_d \cos^2\alpha + \varepsilon_r \sin^2\alpha \tag{7}$$

$$\varepsilon_t = \varepsilon_d \sin^2\alpha + \varepsilon_r \cos^2\alpha \tag{8}$$

$$\frac{\gamma_{lti}}{2} = \left(-\varepsilon_d + \varepsilon_r\right)\sin\alpha\cos\alpha \tag{9}$$

$$\theta = \frac{P_o}{2A_o\gamma_{lti}} \tag{10}$$

$$\psi = \theta \sin 2\alpha \tag{11}$$

$$t_d = \frac{\varepsilon_{ds}}{\psi} \tag{12}$$

$$\varepsilon_d = \frac{-\varepsilon_{ds}}{2} \tag{13}$$

where $\varepsilon_l$, $\varepsilon_t$, and $\gamma_{lt}$ = three strain components in the *l*, *t*-coordinate; $\varepsilon_d$ and $\varepsilon_r$ = strains in the *d*- and *l*-

directions, respectively; $\theta$ = angle of twist of a member; $p_0$ = perimeter of the center line of shear flow zone; $\phi$ = bending curvature of concrete struts; and $\varepsilon_{ds}$ = maximum strain of concrete struts.

For a multiple cell box under pure torsion, a set of simultaneous equations for cell $i$ was simplified as

$$\frac{\gamma_{lti}}{2} = \left(-\varepsilon_{di} + \varepsilon_{ri}\right)\sin\alpha_i \cos\alpha_i \tag{14}$$

$$\theta = \theta_i = \frac{p_{0i}}{2A_{0i}}\gamma_{lti} \tag{15}$$

$$\psi_i = \theta \sin 2\alpha_I \tag{16}$$

**Constitutive Laws of Materials**

The solution of the preceding equilibrium and compatibility equations requires six stress-strain relationships for concrete strut in compressions ($\sigma_d$ to $\varepsilon_d$), and tension ($\sigma_r$ to $\varepsilon_r$), mild steel in longitudinal direction ($f_l$ to $\varepsilon_l$), and transverse direction ($f_t$ to $\varepsilon_t$), and prestressing steel in longitudinal direction ($f_{lp}$ to $\varepsilon_{p}$), and transverse direction ($f_{lp}$ to $\varepsilon_{p}$). General expressions for the constitutive laws of concrete and steel are as follows:

Concrete Struts:

$$\sigma_d = k_1 \zeta f'_c \tag{17}$$

$$k_1 = \xi_1 f_1\left(\varepsilon_{ds}, \zeta\right) \tag{18}$$

$$\zeta = \xi_2\left(\varepsilon_d, \varepsilon_r\right) \tag{19}$$

$$\sigma_r = 0 \tag{20}$$

Steel:

$$f_l = \xi_3\left(\varepsilon_l\right) \tag{21}$$

$$f_t = \xi_4\left(\varepsilon_t\right) \tag{22}$$

where $k_1$ = coefficient defined as the ratio of the average compressive stress $\sigma_d$ to the softened compressive peak stress $\sigma_p = \zeta f'_c$ in concrete struts; $\zeta$ = softening coefficient; and $f_c$ = concrete strength.

For a multiple cell box, the equations are modified to the following:

Concrete Struts:

$$\sigma_{di} = k_{1i} \zeta_i f'_c \tag{23}$$

$$k_{1i} = \xi_1\left(\varepsilon_{dsi}, \zeta_i\right) \tag{24}$$

$$\zeta_i = \xi_2\left(\varepsilon_{di}, \varepsilon_{ri}\right) \tag{25}$$

Steel:

$$f_{li} = \xi_3\left(\varepsilon_{li}\right) \tag{26}$$

$$f_{ti} = \xi_4\left(\varepsilon_{ti}\right) \tag{27}$$

**Selected Constitutive Equations**

For the treatment of torsion, the constitutive equations, Equations [(23)-(27)] will be selected in the following text. The simple elastic perfectly plastic stress-strain relationship of bare mild-steel bars was assumed, because the tensile stress of concrete has been neglected. Parabolic curves were assumed for both concrete and prestressing steel. The softening concrete stress-strain curve proposed by Hsu et al. (1991a, b) is adopted in this study. A softening coefficient $\zeta$ is used to describe the ascending [(28)] and descending [(29)] branches of the curve. The stress-strain curve of prestressing steel consists of one straight line up to $0.7f_{pu}$ and the second part expressed by Ramberg-Osgood equation.

Concrete Struts:

$$k_{1i} = \frac{\varepsilon_{dsi}}{\zeta_i \varepsilon_0}\left(1 - \frac{1}{3}\frac{\varepsilon_{dsi}}{\zeta_i \varepsilon_0}\right), \qquad \frac{\varepsilon_{dsi}}{\varepsilon_p} \leq 1 \tag{28}$$

$$= \frac{\zeta_i^2}{(2-\zeta_i)^2}\frac{\varepsilon_{dsi}}{\zeta_i \varepsilon_0}\left(1 - \frac{1}{3}\frac{\varepsilon_{dsi}}{\zeta_i \varepsilon_0}\right), \qquad \frac{\varepsilon_{dsi}}{\varepsilon_p} > 1 \tag{29}$$

$$\zeta_i = \frac{0.9}{\sqrt{1 + 600\varepsilon_{ri}}} \tag{30}$$

Mild Steel:

$$f_l = E_s \varepsilon_l \qquad \varepsilon_l < \varepsilon_{ly} \tag{31}$$

$$f_l = f_{ly} \qquad \varepsilon_l \geq \varepsilon_{ly} \tag{32}$$

$$f_t = E_s \varepsilon_t \qquad \varepsilon_t < \varepsilon_{ty} \tag{33}$$

$$f_t = f_{ty} \qquad \varepsilon_t \geq \varepsilon_{ty} \tag{34}$$

Prestressing Steel:

$$f_p = E_{ps}\left(e_{dec} + e_s\right), \qquad f_p \leq 0.7 f_{pu} \tag{35}$$

$$f_p = \frac{E'_{ps}\,(e_{dec} + e_s)}{[1 + \{(E'_{ps}\,(e_{dec} + e_s)) / f_{pu}\}^m]^{1/m}}, \qquad f_p > 0.7 f_{pu} \tag{36}$$

where $\varepsilon_0$ = strain at $f_c$ usually taken as 0.002; $\varepsilon_p$ = compressive peak strain in concrete struts; $E_s$ = modulus of elasticity of steel bars; $\varepsilon_{ly}$ and $\varepsilon_{ty}$ = yield strains of longitudinal and transverse steel bars, respectively; $f_{ly}$ and $f_{ty}$ = yield stresses of longitudinal and transverse steel bars, respectively; $f_p$ = stress in prestressing steel; ($f_p$ becomes $f_{lp}$ or $f_{tp}$ when applied to the longitudinal and transverse steel, respectively); $e_s$ = strain in prestressing steel; ($e_s$ becomes $e_l$ or $e_t$ when applied to the longitudinal and transverse steel, respectively); $e_{\mathrm{dec}}$ = strain in prestressing steel at decompression of concrete; $E_{\mathrm{ps}}$ = modulus of elasticity of prestressing steel, taken as 2 x $10^5$ MPa (29,000 ksi); $E'_{\mathrm{ps}}$ = tangential modulus of the Ramberg-Osgood curve [taken as 2.14 x $10^5$ MPa or (31,060 ksi)]; $f_{\mathrm{pu}}$ = ultimate strength of prestressing steel; and $m$ = shape parameter (taken as 4).

The strain in prestressing steel at decompression of concrete $e_{\mathrm{dec}}$ is considered a known value and is approximately equal to 0.005 for 1,720-MPa (250-ksi) and 1,860-MPa (270-ksi) prestressing stands.

**Derived Equations**

The thickness of shear flow zone $t_{di}$ can be expressed in terms of strains using the compatibility equations [(7) – (12) and (14) – (16)]. To do this, first combine the four compatibility equations [(9) – (12)] into one equation.

$$t_{di} = \frac{A_{0i}}{p_{0i}} \left[ \frac{(\ - \varepsilon_{di})(\varepsilon_{ri} - \varepsilon_{di})}{(\varepsilon_{li} - \varepsilon_{di})(\varepsilon_{ti} - \varepsilon_{di})} \right] \tag{37}$$

The strain $\varepsilon_l$ can be related to the stress $f_l$ by eliminating the angle $\alpha$ from the equilibrium equation (1)

$$\varepsilon_{li} = \varepsilon_{di} + \frac{A_{0i}(-\varepsilon_{di})(-\sigma_{di})}{(A_{li}\, f_{li} + A_{lpi}\, f_{lpi})} \tag{38}$$

Similarly, the strain $\varepsilon_t$ can be related to the stress $f_t$ by eliminating the angle $\alpha$ from equilibrium equation (2)

$$\varepsilon_{ti} = \varepsilon_{di} + \frac{A_{0i}\, \sigma\, (-\varepsilon_{di})(-\sigma_{di})}{p_{0i}\,(A_{ti}\, f_{ti} + A_{tpi}\, f_{tpi})} \tag{39}$$

Additionally, $A_{0i}$ and $p_{0i}$ can be expressed as functions of $t_{di}$

$$A_{0i} = A_{ci} - \frac{1}{2}\, p_{ci}\, t_{di} + t_{di}^2 \tag{40}$$

$$p_{0i} = p_{ci} - 4\, t_{di} \tag{41}$$

where $A_{ci}$ = cross-sectional area of the $i^{th}$ cell bounded by the outer perimeter of the concrete; $p_{ci}$ = perimeter of the $i^{th}$ cell outer concrete cross section.

The values of $\varepsilon_{ri}$ and $\alpha$ can be expressed in terms of strains $\varepsilon_{li}$, $\varepsilon_{ti}$, and $\varepsilon_{di}$ by

$$\varepsilon_{ri} = \varepsilon_{li} + \varepsilon_{ti} - \varepsilon_{di} \tag{42}$$

$$\tan^2 \alpha_i = \frac{\varepsilon_{li} - \varepsilon_{di}}{\varepsilon_{ti} - \varepsilon_{di}} \tag{43}$$

From (11) – (13) an additional equation is obtained

$$\varepsilon_{di} = -\frac{1}{2}\, t_{di}\, \theta \sin 2\alpha_i \tag{44}$$

**Solution Procedures**

A set of solution Procedures is proposed, as shown in the flow chart of Figure 4. A computer program has been written to analyze the torsional behavior of reinforced concrete and prestressed concrete members with multiple box sections according to the flow chart.

## EXAMPLE AND DATA ANALYSIS

A program was developed for analyzing both reinforced concrete and prestressed concrete structures. This computer program is based on the **Softened Truss Model** theory and is used to compare the results developed based on the **Theory of Elasticity**, which is commonly used by structural engineers.

**Softened Truss Model Theory Vs. Simplified Elastic Solution**

The data of this example are from Heins and Lawrie (1984, example 7.1) for a bridge design with a section of the multicontinuous cells. The bridge to be designed is shown in Figure 5 in sections near midspan and at the pier. But, for the torsional design, one only uses the section near the pier. The material strengths are $f_c'$ = 27.6 MPa (4,000 psi) and $f_y$ = 414 MPa (60,000 psi). The total applied torques $T$ are 6,230, 12,680, and 18,690 m-KN (55,140, 112,230 and 165,420 in.-kip), respectively. After using the analysis program to calculate the example, the results are compared with example from Heins and Lawrie (Tables 1 – 3). By comparing the results from analysis and Heins and Lawrie (1984), one notices that the differences vary according to the different values of $T$. This happens because Heins and Lawrie (1984) use the simplified elastic solution with the assumption that a section is thin walled and thus has a uniform shear flow across its thickness. This is usually true when a section is made of steel. But walls made of concrete are much thicker than those made of steel. Based on the Theory of Elasticity, the equation is

$$q_i = \tau_i h_i = \left(\frac{C_i}{K_T}\right) T \tag{45}$$

or

$$\tau_i = \left(\frac{C_i}{K_T h_i}\right) T \tag{46}$$

Equation (46) does not consider the softening behavior of reinforced concrete, and $\tau_i$ is independent of the concrete strength $f_c'$. So, for a given wall thickness $h_i$, $C_i/(K_T h_i)$ is a constant, and the relationship between $\tau_i$ and $T$ is linear. Actually, full wall thickness, $h_i$ should not be used. Instead, equivalent thickness of the shear flow zone, $t_{di}$, according to the softened truss model theory, should be used. Hence, we should use a formula as follows

$$\tau_i = \left(\frac{C_i}{K_T t_{di}}\right) T \tag{47}$$

where $t_{di}$ = a variable instead of constant, thus the relationship is not linear between $\tau_1$ and $T$. To explain clearly the relationship between $t_{di}$ and $T$, one may observe the simplified formula for a single cell box section given by Hsu (1993)

$$t_d = \frac{4T}{A_c\ f'_c} \tag{48}$$

For a given cross section, the relationship between $t_d$ and $T$ is proportional and between $t_d$ and $f'_c$ is inversely proportioned. When $f'_c$ is given, the relationship between $t_d$ and $T$ is nearly linear. That means that when the value of $T$ increases, the value of $t_d$ increases much faster than the value of $\tau$. This can be observed from Tables 1 – 3. When the theory of elasticity is applied, because $h$ is a constant, the value of $\tau$ will vary linearly with $T$. When the Softened Truss Model Theory is used, because $t_d$ is a variable, the value of $t_d$ will vary almost linearly with $T$ while $\tau$ varies very slowly. From the tables, it can be seen that when $T$ = 6,230 m-KN (55,140 in.-kip), comparing the softened truss model method and the method of elastic theory, the maximum differences $e_{max}$ = 55% for $\tau$. The elastic theory is unconservative because $t_d$ is much smaller than $h_{min}$. When $T$ = 18,690 m-KN (165,420 in.-kip), the $e_{max}$ = 53% for $\tau$ and the elastic theory is conservative because $t_{d3}$ is much larger than $h_{min}$. When $T$ = 12,680 m-KN (112,230 in.-kip), $t_{d3}$ = $h_{min}$ = 200 mm (8 in.) for cell 3, and difference $e_3$ only equals 5% for $\tau_3$. So, one realizes that only when $T$ gets some special value and then the value of $t_{di}$ closes the value of $h_i$, one can get approximately the same $\tau_i$. Because the theory of elasticity does not consider the softening behavior of reinforced concrete and uses a constant $h$ instead of a variable $t_d$, $\tau$ may have a very large value when $T$ increases. This is not true according to the softened truss model theory. When the concrete strut cracking angle $\alpha$ is given, the shear stress $\tau$ depends only on the softened compressive strength of concrete struts $\sigma_d$ and can vary only through a small range. The resistance mainly depends on the variation of $t_d$. This characteristic of reinforced concrete can be used to determine sizes of cross sections of reinforced concrete bridges. In fact, from the Table 3, when $T$ = 18,690 m-KN (165,420 in.-kip), the value of $t_d$ is larger than the actual flange thickness $h_{min}$ computed by the softened truss model method. The flange thickness has to be increased to get enough resistance.

## Data Analysis Based on Softened Truss Model Theory

The same example as shown in Figure 5 was used with the following alternations and results are shown in Table 4 of this paper:

1. Data with prestressing percentage $\mu_l = \mu_t = 0.0$ (no prestressing)
2. Data with prestressing percentage $\mu_l = 0.35$, $\mu_t = 0.0$ (prestressing longitudinally only)
3. Data with prestressing percentage $\mu_l = 0.35$, $\mu_t = 0.35$ (prestressing both longitudinally and transversely)

By inspection, the following conclusions for these various cases can be made.

From Data 1:

- If prestressing steel is not used, $\mu_l = \mu_t = 0.0$, and then the exactly same results as that shown by Fu and Yang (1996) for reinforced concrete case can be obtained.
- Torque based on single cell assumption is higher (24.1%) than the total torque of all cells based on multi-cell assumption.

From Data 2:

- When prestressing only in longitudinal direction is considered, $\mu_l = 0.35$, and $\mu_t = 0.0$, the torsional capacity $T$ increases and twist angle $\theta$ decreases.
- Because the longitudinal direction stiffness has been increased, angle $\alpha$ decreases.
- Torque based on single cell assumption is slightly higher (5.5%) than the total torque of all cells based on multi-cell assumption

From Data 3:

- When the same percentage prestressing in both directions is used, $\mu_l = \mu_t = 0.35$ and angle $\alpha$ comes back to 45º.
- This program gives good results when the section reaches its capacity ($\varepsilon_d > 0.0018$).
- Torque based on single cell assumption is lower (12.6%) than the total torque of all cells based on multi-cell assumption

## SUMMARY AND CONCLUSIONS

- Usually, bridge engineers use the Theory of Elasticity to deal with torsional problems in bridge design. Basically this is true for steel bridges with sections of multiple box cells because steel structures meet the assumption that the sections are thin walled and thus have a uniform shear flow across their thickness. Concrete bridges, however, usually have much thicker walls than steel bridges, and do not meet the thin wall condition. Actually, according to the unified theory of reinforced concrete, the shear flow zone for a boxed section has a thickness denoted $t_d$. This thickness $t_d$ is a variable determined from the equilibrium and compatibility conditions. It is not the same as the given wall thickness $h$.
- The Theory of Elasticity cannot consider the softened effect due to concrete behavior. The softened truss model emphasizes the importance of incorporating the softened constitutive laws of concrete in the analysis of reinforced concrete structures.
- The new method and the computer flow chart presented in this article have been developed according to the softened truss model theory, which was first mentioned by Hsu (1988). In this theory, the concrete torsional problem was solved theoretically for the first time by combining equilibrium conditions and compatibility conditions and constitutive laws of materials. However, up to now the theory has been only applied to the case of pure torsion with single cell section. Experimental data for single cell were also presented in the same reference.
- Based on the Softened Truss Model Theory, a method has been developed by Fu and Yang (1996) to deal with the torsional problem, especially for reinforced concrete-box girder bridge superstructures with multiple cell sections. By adding some new equations, this method has been made applicable to both reinforced concrete and prestressed concrete structures.
- Results for single cell case by using the new algorithm are compared with previous theoretical and experimental work done by Hsu (1993) and Fu and Yang (1996). The multicell cases are then illustrated by numerical examples and compared with single cell assumptions.

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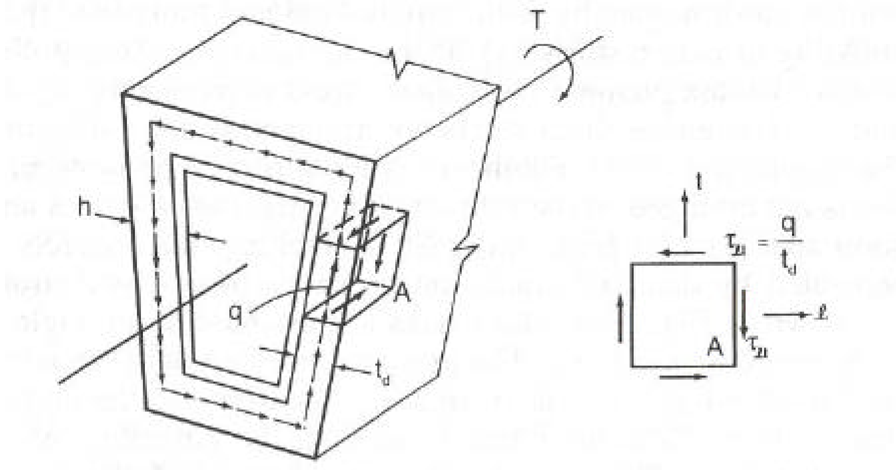



(a) Shear Flow in an Element (b) Shear Stress on Element A

**FIG. 1. Hollow Box Subjected to Torsion**

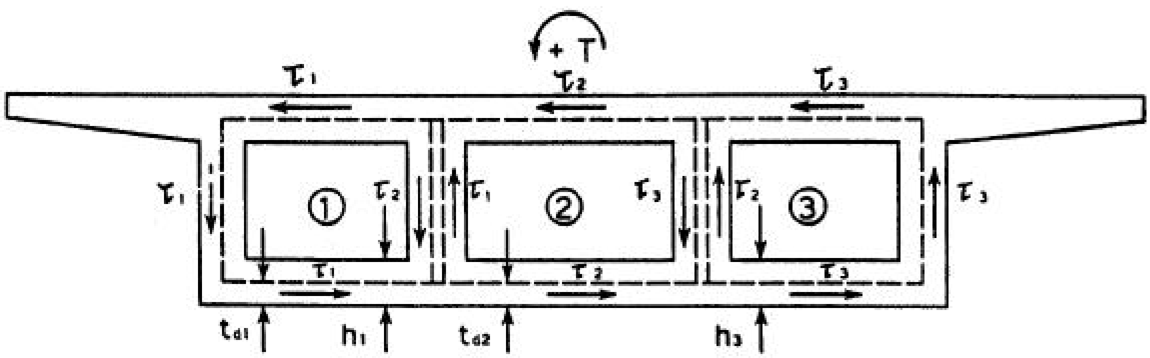
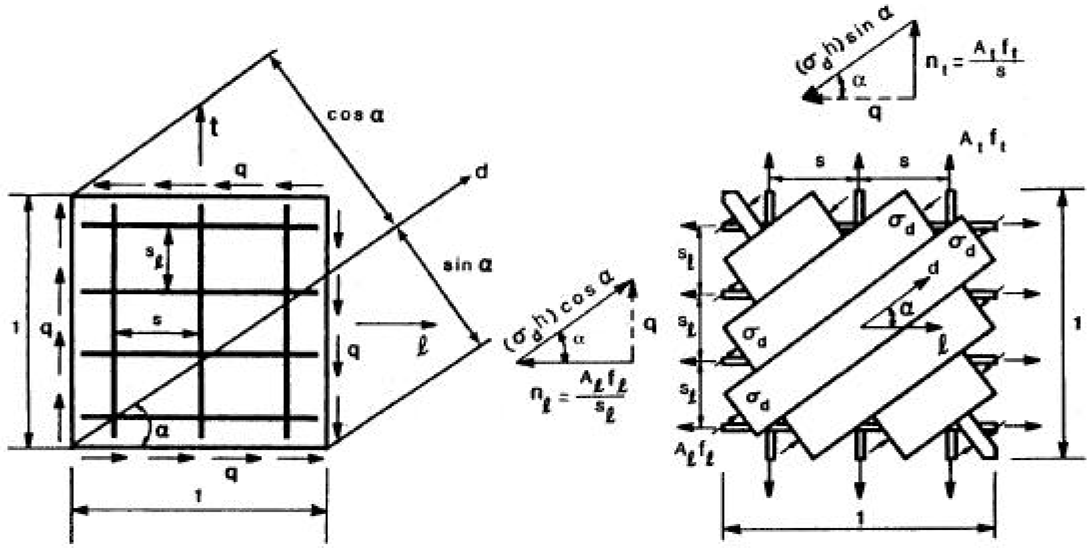



**FIG. 2. Shear Stresses in Multicell Section**




(a) Shear Element (b) Truss Element

**FIG. 3. Equilibrium in Element Shear**

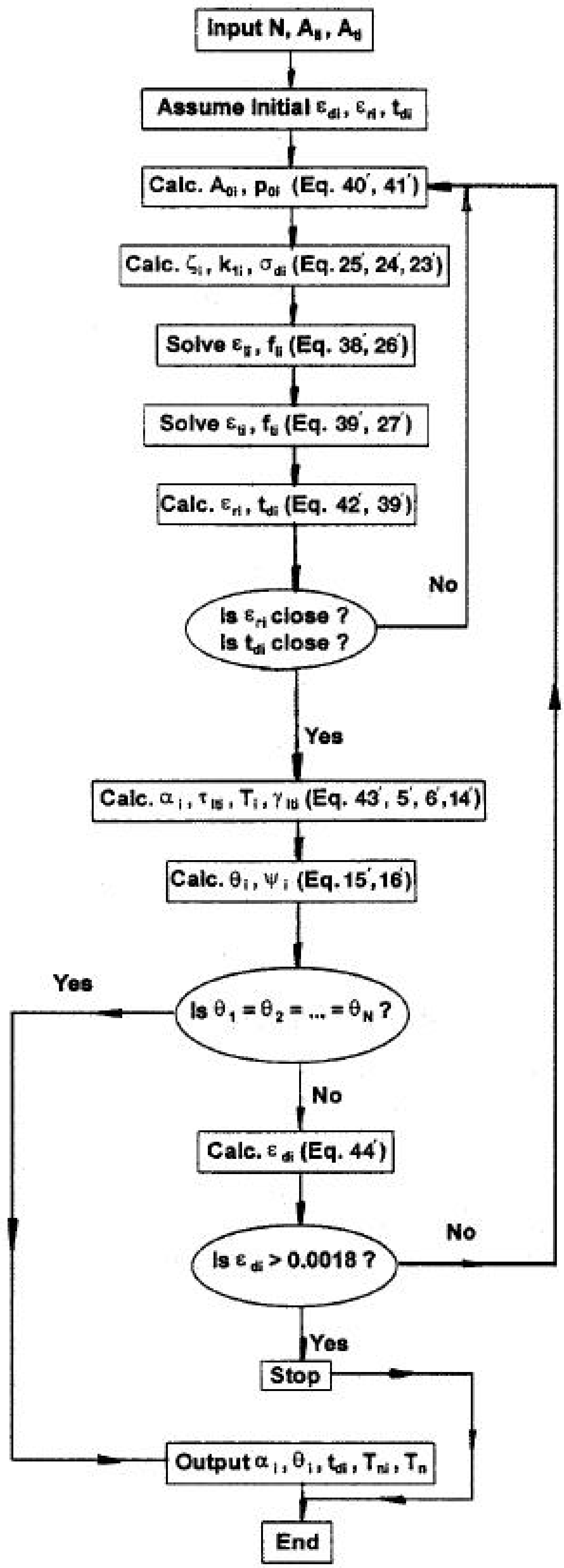



**FIG. 4. Flowchart for Torsional Analysis**

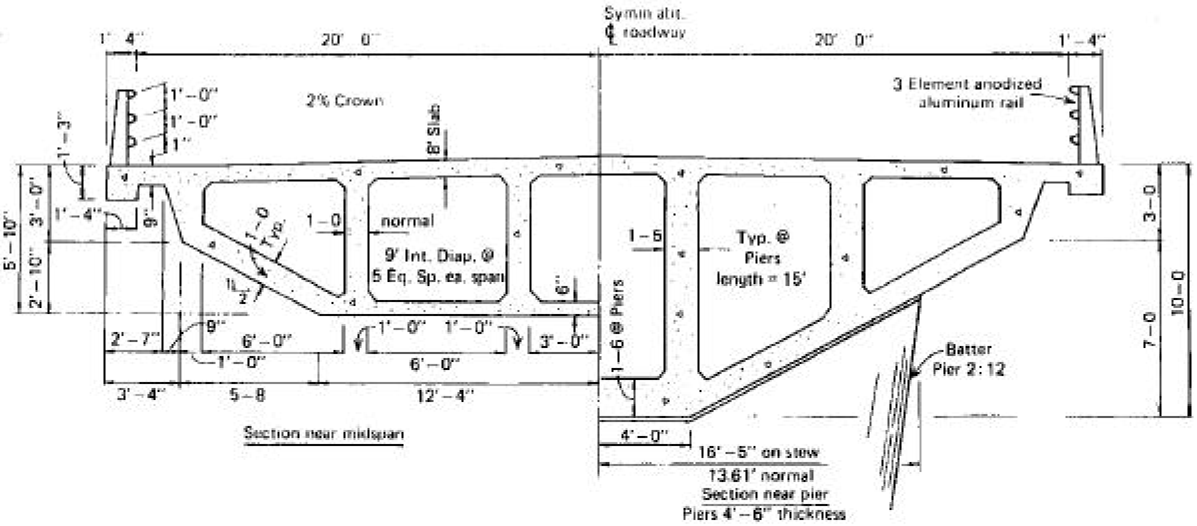



**FIG. 5. Typical Bridge Example under Torsion**

**Table 1 - Data Analysis (T = 6,230 m-KN or 55,140 in-kips)**

| T= 6,230 m-KN (55,140 in.-kip) | Cell No | $t_{di}$ [mm (in.)] | $\hat{o}_i$ [MPa (psi)] | Difference for $\hat{o}_i$ (%) | Compare |
|---|---|---|---|---|---|
| Proposed Analysis | 1 | 46 (1.81) | 1.0 (144.94) | --- | --- |
| | 2 | 78 (3.09) | 1.64 (237.36) | --- | --- |
| | 3 | 95 (3.73) | 1.92 (278.63) | --- | --- |
| Design in Heins & Lawrie (1984) | 1 | $h_{min}$ = 203 (8.0) | 0.48 (69.25) | 52.00% | Analysis (Heins and Lawrie 1984) |
| | 2 | $h_{min}$ = 203 (8.0) | 0.73 (106.50) | 55.00% | |
| | 3 | $h_{min}$ = 203 (8.0) | 1.02(147.38) | 47.00% | |

**Table 2 - Data Analysis (T = 12,680 m-KN or 112,230 in-kips)**

| T= 12,680 m-KN (112,230 in.-kip) | Cell No | $t_{di}$ [mm (in.)] | $\hat{o}_I$ [MPa (psi)] | Difference for $\hat{o}_i$ (%) | Compare |
|---|---|---|---|---|---|
| Proposed Analysis | 1 | 100 (3.92) | 1.01 (146.71) | --- | --- |
| | 2 | 170 (6.68) | 1.67 (242.37) | --- | --- |
| | 3 | 203 (8.00) | 1.97 (285.18) | --- | --- |
| Design in Heins & Lawrie (1984) | 1 | $h_{min}$ = 203 (8.0) | 0.97 (140.95) | 4.00% | Analysis (Heins and Lawrie 1984) |
| | 2 | $h_{min}$ = 203 (8.0) | 1.49 (216.77) | 11.00% | |
| | 3 | $h_{min}$ = 203 (8.0) | 2.07 (299.97) | -5.00% | |

**Table 3 - Data Analysis (T = 18,690 m-KN or 165,420 in-kips)**

| T= 18,690 m-KN (165,420 in.-kip) | Cell No | t$_{di}$ mm (in.) | ô$_i$ MPa (psi) | Difference for ô$_i$ (%) | Compare |
|---|---|---|---|---|---|
| Proposed Analysis | 1 | 158 (6.23) | 1.01 (146.72) | --- | --- |
| | 2 | 272 (10.69) | 1.69 (244.90) | --- | --- |
| | 3 | 325 (12.80) | 1.99 (288.92) | --- | --- |
| Design in Heins & Lawrie (1984) | 1 | $h_{min}$ = 203 (8.0) | 1.43 (207.75) | -42.00% | Analysis (Heins and Lawrie 1984) |
| | 2 | $h_{min}$ = 203 (8.0) | 2.20 (319.50) | -31.00% | |
| | 3 | $h_{min}$ = 203 (8.0) | 3.05 (442.14) | -53.00% | |

[a]*T* = 18,690 m-kN (165,420 in. –kips).

**Table 4 - Analysis Comparison of Various Longitudinally and Transversely Prestressing Percentages**

| Prestressing percentage (1) | Case (2) | Cell number (3) | á$_i$ (degrees) (4) | è$_i$ [rad/mm (rad/in.)] (5) | ô$_{lti}$ [MPa (psi)] (6) | T$_i$ [kN-m (in.-kips)] (7) | Total torque [kN-m (in.-kips)] (8) | Difference (%) (9) |
|---|---|---|---|---|---|---|---|---|
| ì$_l$ = 0 | Five cell | 1 | 45 | $0.153 \times 10^{-5}$ ($0.389 \times 10^{-4}$) | 1,011 (146.7) | 616 (5,453) | 12,672 (112,158) | ---- |
| ì$_t$ = 0 | | 3 | 45 | $0.153 \times 10^{-5}$ ($0.389 \times 10^{-4}$) | 1,966 (285.2) | 5,370 (47,530) | | |
| | Single cell | 1 | 51.81 | $0.175 \times 10^{-5}$ ($0.445 \times 10^{-4}$) | 1,896 (275.0) | 15,726 (139,194) | 15,726 (139,194) | +24.10 |
| ì$_l$ = 0.35 | Five cell | 1 | 26.35 | $0.854 \times 10^{-6}$ ($0.217 \times 10^{-4}$) | 859 (124.6) | 1,049 (9,286) | 18,180 (160,916) | ---- |
| ì$_t$ = 0 | | 3 | 29.66 | $0.845 \times 10^{-6}$ ($0.217 \times 10^{-4}$) | 1,603 (232.5) | 7,289 (64,514) | | |
| | Single cell | 1 | 42.92 | $0.122 \times 10^{-5}$ ($0.310 \times 10^{-4}$) | 1,842 (267.12) | 19,179 (169,757) | 19,179 (169,757) | +5.50 |
| ì$_l$ = 0.35 | Five cell | 1 | 44.58 | $0.748 \times 10^{-6}$ ($0.190 \times 10^{-4}$) | 4,624 (670.6) | 4,254 (37,656) | 58,583 (518,529) | ---- |
| ì$_t$ = 0.35 | | 3 | 43.63 | $0.748 \times 10^{-6}$ ($0.190 \times 10^{-4}$) | 3,692 (535.4) | 20,391 (180,481) | | |
| | Single cell | 1 | 56.68 | $0.066 \times 10^{-4}$ ($0.167 \times 10^{-3}$) | 3,767 (54,628) | 51,211 (453,276) | 51,211 (453,276) | – 12.6 |